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THERMAL INTERACTION OF HIGH-POWER LASER RADIATION WITH GASES

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Abstract—The paper is concerned with the study of thermal interaction of radiation with a quiescent medium and with a medium in co- and counter-current flow over the path being traversed by radiation. Operation of a gas lens focusing high-power lasing in the case when radiation-induced heat generation contributes additionally to temperature nonuniformity is considered. The conditions required for self-focusing of stationary Gaussian and circular beams the intensity of which decreases towards the beam center are studied. Dynamics of thermal and optical processes in the case of "banana" self-focusing is considered.

NOMENCLATURE

x, y, z,	coordinates;
x,	direction of radiation propagation;
t,	time;
$\nabla_\perp$,	$= \hat{y}\partial_y + \hat{z}\partial_z$, $\hat{y}$ and $\hat{z}$ being coordinate unit vectors;
T_0,	temperature of a homogeneous medium;
ϑ,	nonuniformity of the medium temperature in the absence of radiation;
Θ,	nonuniformity of the gas temperature due to radiation absorption;
P,	pressure;
ρ,	density of the medium;
c_v, c_p,	heat capacity at constant volume and pressure;
ν,	kinematic viscosity;
λ,	thermal conductivity;
a,	thermal diffusivity;
k,	radiation wave vector;
λ_0,	radiation wavelength;
c,	speed of light;
E,	electric field strength in radiation beam;
I,	radiation intensity;
ε,	dielectric constant of the medium;
ε_T,	$= \frac{d\varepsilon}{dT}$ at $T = T_0$;
α,	radiation absorption coefficient;
$\Lambda_\perp$,	halfwidth of radiation beam;
$\Lambda_\parallel$,	characteristic scale of radiation intensity change in direction x (focusing or defocusing length);
Pe, Fo, Ho,	Peclet, Fourier and homochromaticity number, respectively.

Subscript

$\perp$,	transverse component with respect to direction of radiation propagation x.

1. INTRODUCTION

THE LIGHT rays in a homogeneous medium are bent into regions of higher refractive index – hence the refraction of light. The refractive indices of gases decrease with increasing temperature. This is the reason why the heating produced by absorbed radiation leads to defocusing of high-power laser beams thus being responsible, among other things, for a constraint on radiative power which can be transmitted efficiently through the atmosphere or through the medium of a laboratory or industrial setup. Thermal blooming of radiation is characterized by a low threshold amount of energy and can occur at radiation intensities on the order of 1 W even in the atmospheric windows which are distinguished by low absorptivity [1–3]. For long enough pulses and also for stationary and quasi-stationary radiation, heating is the main reason for a constraint on the output power [4, 5].

Intense radiation propagating in a gas gives rise to a variety of interesting phenomena. Thus, in a quiescent gas it generates photoabsorption convection which, at sufficiently high intensities, leads to turbulent flow of the medium [3, 6, 7]. This type of convection produces distortions in distribution of radiation intensity over the beam [8]. In a medium moving crosswise, the beam deflects to the direction of motion and can even split into several beams [2, 3, 5, 8, 9]. The effect of "kinetic" cooling of the medium in a CO_2-laser beam is possible [2, 10].

Much work, both experimental and theoretical, has been done to date on the problem of thermal interaction of radiation with gases. The majority of publications have been reviewed in [1–3, 11]. Most of the studies concern thermal interaction of radiation with a quiescent medium and a medium moving across the beam. Correlations have been established between deflection of a beam as a whole and distortion of the beam cross-section, on the one hand, and the heat transfer regime and radiation parameters, on the other.

In particular, it has been shown that beam behaviour depends strongly on the time of radiant flux interaction with the medium, say, on the pulse length and radiation intensity distribution over the beam. Thus, the output pulses which have a circular intensity profile focus at the center and defocus at the edges (the so-called "banana" self-focusing [12, 13]), those with a Gaussian intensity profile defocus. Stationary radiation in a quiescent medium defocuses irrespective of the form of the intensity distribution [14–16]. In a medium moving across the beam, the beams with any intensity distribution become focused but the process is accompanied by beam deflection and splitting into several beams and this decreases the efficiency of focusing [5, 9].

Problems related to compensation for thermal distortion by means of adaptive optics and a proper choice of the initial focusing of the beam, etc. are discussed in [2, 3, 11].

However, a large number of problems have been studied in less detail. This concerns thermal interaction of stationary radiation with a co- and counter-current gas flows and propagation of high-power radiation pulses in shear flows. Thus the possibilities for the radiation transfer processes to be controlled thermooptically by selecting proper conditions for thermal interaction of radiation with a gas have not been used to full advantage.

In the present work, the parameters have been determined which make for minimal radiation losses in transit by a proper choice of the conditions for thermal interaction of laser beams with gases. The following three problems are considered: (1) thermal interaction of stationary and quasi-stationary radiation beams in co- and counter-current gas flows; (2) the effect of heat transfer on propagation of high-power radiation pulses in shear flows (gas lenses); (3) thermal interaction of radiation pulses of circular beams in quiescent gases.

2. STATEMENT OF THE PROBLEM

Let us consider the ranges of variation of the problem parameters which are typical of thermal interaction of radiation with gases. The gas flow velocities are much below the speed of sound. The radiative flux density, $E^2/8\pi$, is much smaller than the internal energy density, i.e.

$$I \lesssim c\rho c_v T. \quad (1)$$

In the atmospheric air under normal conditions $c\rho c_v T_0 \approx 10^{14}\ \mathrm{W/m^2}$. A change in the gas temperature, Θ, is smaller than the gas temperature with no radiation, i.e. $\Theta \lesssim T_0$. Under atmospheric conditions Θ can amount to tens of degrees. For the condition (1) to be satisfied, it is also assumed that the intensities are far from those of the breakdown threshold. Nonuniformity of the gas temperature in the absence of radiation, ϑ, is also relatively small. Therefore, the physical properties of the medium and the radiation absorption coefficient, α, are assumed to be constant. Characteristic times of radiation interaction with the medium (say, the radiation pulse length τ_i) greatly exceed the mean free path time of the gas molecules and also the time of propagation of the acoustic disturbances ρ and T. All of the scales of nonuniformities of the problem parameters are much above the free path length of the gas molecules and the radiation wavelength λ_0.

Under such equilibrium conditions, thermal interaction of radiation with the medium dominates the process, the dielectric permittivity of the medium (the refractive index being $\sqrt{\varepsilon}$) is the function of the temperature alone, and, due to a relative smallness of ϑ and Θ, is of the form [4, 5]:

$$\varepsilon(T) = \varepsilon_0 + \varepsilon_T(\vartheta + \Theta) + i\varepsilon'', \quad (2)$$

where

$$\varepsilon_T = \left.\frac{d\varepsilon}{dT}\right|_{T=T_0} < 0,\ \varepsilon_0, \varepsilon'' = \mathrm{const} > 0$$

and

$$|\varepsilon_T(\vartheta + \Theta)|, \varepsilon'' \ll \varepsilon_0.$$

The imaginary part of ε defines absorption of radiation and is related to the absorption coefficient as $\alpha = \varepsilon'' k/\varepsilon_0$. For optical radiation in the atmospheric windows $\varepsilon_0 = 1.00029$, $a \lesssim 10^{-4}\ \mathrm{m^{-1}}$, $\varepsilon_T = -2.3 \cdot 10^{-6}\ \mathrm{K^{-1}}$.

Fluid motion and heat transfer are described by the system of Navier–Stokes and energy equations which is solved jointly with the system of Maxwell equations. However, under the conditions assumed the latter system can be transformed into a single, more simple, parabolic-type equations (5) (quasi-optical approximation [17]) and the equations for the problem take on the form [5, 7, 9, 11]:

$$\partial_t \mathbf{V} + (\mathbf{V}\nabla)\mathbf{V} = -\frac{\nabla P}{\rho} + \nu\nabla^2\mathbf{V} + g\beta(\vartheta + \Theta), \quad (3)$$

$$\operatorname{div} \mathbf{V} = 0, \quad (4)$$

$$\rho c_p[\partial_t(\vartheta + \Theta) + \mathbf{V}\nabla(\vartheta + \Theta)] = \lambda\nabla^2(\vartheta + \Theta) + \frac{\alpha\sqrt{\varepsilon_0}c}{8\pi}|E|^2, \quad (5)$$

$$-2ik\partial_x E = \nabla_\perp^2 E + \frac{k^2}{\varepsilon_0}[\varepsilon_T(\vartheta + \Theta) + i\varepsilon'']E, \quad (6)$$

in which the second term on the RHS of (5) is αI, and the radiation intensity, I, is of the form

$$I = \frac{c\sqrt{\varepsilon_0}}{8\pi}|E|^2. \quad (7)$$

Equations (3)–(5) are written in the Boussinesq approximation. In a forced flow, the last term on the RHS of equation (3) will not be taken into account.

The heat source on the RHS of (5) is due to radiation absorption. Equation (6) describes propagation of radiation beams of a finite width allowing for the fact that $\Lambda_\perp \gg \Lambda_\parallel$.

The solution of the heat-transfer problem subject to the prescribed boundary and initial conditions in the absence of radiation is assumed to be known as well as the specific parameters for $\mathbf{V}$ and ϑ.

The initial and boundary conditions for Θ are specified for each particular case, since beams are considered far from the gas-confining surfaces. Therefore at $y, z \to \infty$ the function Θ and its lateral spatial derivatives tend to zero.

The boundary conditions for E are

$$E^2(x = 0, y, z) = E_0^2 \exp\left(-\frac{y^2 + z^2}{\Lambda_\perp^2}\right) f(t), \quad (8)$$

or

$$E^2(x = 0, y, z) = E_0^2 \left(\frac{y^2 + z^2}{\Lambda_\perp^2}\right) \exp\left(-\frac{y^2 + z^2}{\Lambda_\perp^2}\right) f(t), \quad (9)$$

$$\partial_y E, \partial_z E, E\Big|_{y,\, z \to \infty} \to 0,$$

i.e. the most practically interesting cases of Gaussian (8) and circular (9) beams are considered, with the dimensionless function $f(t)$ specifying the radiation pulse shape in time. In equation (6), time is a parameter of the problem.

The heat source in equation (4) is of a peculiar type which imparts a specific nature to heat transfer. First, emission of radiation entails no changes in the boundary conditions in the sense that no additional surfaces are introduced into the gas flow which is free to move through a radiative flux. Second, the heat source – a laser beam – is spatially very inhomogeneous [see (8), (9)]. The velocity profile of the radiation-induced flow has therefore the inflexion points and, with no walls to stabilize the flow, the latter is very unstable, i.e. it is distinguished by very small critical Reynolds and/or Grashof numbers that define transition to a turbulent flow. Turbulence is assumed not to occur; corresponding estimates of the hydrodynamic criteria and critical radiation intentisites responsible for the development of turbulence in various situations are given in [7].

A nonanalytic nonlinearity of the heat source in (5) is due to complex solutions available for equation (6). This singularity is commonly circumvented by seeking the solution of (6) in the amplitude (ψ) – phase (S) variables [17], i.e. by expressing E as

$$E(x, y, z, t) = \psi(x, y, z, t) \exp\{ikS(x, y, z, t)\}. \quad (10)$$

Substitution of (10) transforms (6) into the following system of equations for ψ and S:

$$2\partial_x S + (\nabla_\perp S)^2 = \frac{\varepsilon_T(\vartheta + \Theta)}{\varepsilon_0} + \frac{\nabla_\perp^2 \psi}{k^2 \psi}, \quad (11)$$

$$\partial_x \psi^2 + \nabla_\perp \psi^2 \cdot \nabla_\perp S + \psi^2 \nabla_\perp^2 S + \alpha\psi^2 = 0, \quad (12)$$

where $\alpha = k\varepsilon''/\varepsilon_0$. Now the heat source in the energy equation (5) takes on the form

$$\alpha I = \frac{\alpha c\sqrt{\varepsilon_0}\psi^2}{8\pi}. \quad (13)$$

The normals to the surface S = const determine the trajectories of beams. The beams are directed towards a medium with higher optical density, i.e. towards lower temperatures, and they focus or defocus. The first term on the RHS of (11) stands for warping (refraction) of beams due to the medium temperature nonuniformity; the second term stands for diffraction divergence of beams. As a rule this term (being on the order of $\lambda_0/\Lambda_\perp \ll 1$) is negligible as compared with the first one, and refraction dominates the process. Then the approximate solution of equation (11) for plane and/or axisymmetric beams propagating along the axis x will yield the angle of inclination of beams to this axis [4]:

$$\varphi^2 = \varphi_e^2 + \frac{\varepsilon_T}{\varepsilon_0}\{(\vartheta + \Theta) - (\vartheta + \Theta)_e\}$$

$$= \frac{\varepsilon_T}{\varepsilon_0}\{(\vartheta + \Theta) - (\vartheta + \Theta_e)\}, \quad (14)$$

$$\varphi = \partial_y S,$$

where φ_e is the initial angle hereafter assumed to be zero; ϑ_e, Θ_e are the corresponding temperature nonuniformities at the starting point of the beam trajectory. Since $\varepsilon_T < 0$, $\varphi^2 \geqslant 0$, the beams are directed to a colder region.

Thus, the problem of thermal interaction of radiation with a gas has reduced to the solution of equations (3)–(5), (11), (12).

3. STATIONARY AND QUASI-STATIONARY RADIATION IN A MOVING MEDIUM

For a forced flow of a homogeneous gas along the radiation beam at the velocity $\mathbf{V} = \{V, 0, 0\}$ the heat-transfer, (3)–(5), and radiation, (11), (12), equations take on the form

$$V\partial_x\Theta - a\nabla^2\Theta = \frac{\alpha I}{\rho c_p}, \quad (15)$$

$$2\partial_x S + (\nabla_\perp S)^2 = \frac{\varepsilon_T \Theta}{\varepsilon_0} + \frac{\nabla_\perp^2 I^{1/2}}{k^2 I^{1/2}}, \quad (16)$$

$$\partial_x I + \nabla_\perp I \cdot \nabla_\perp S + I\nabla_\perp^2 S + \alpha I = 0. \quad (17)$$

In the absence of radiation the gas is homogeneous, hence $\vartheta = 0$.

(A) *One-dimensional problem*

In the one-dimensional case, when all of the variables of the problem depend on x alone, the system of equations (15)–(17) allows an exact solution for a beam of an infinite width. For two different directions of velocity V($V = V^{(+)}$ for a co-current flow and $V = -V^{(-)}$ for a counter-current flow), there are two different solutions:

$$\Theta^{(+)} = \frac{I_0\{1 - \exp(-\alpha x)\}}{\alpha\lambda(1 + Pe^{(+)})}, \quad (18)$$

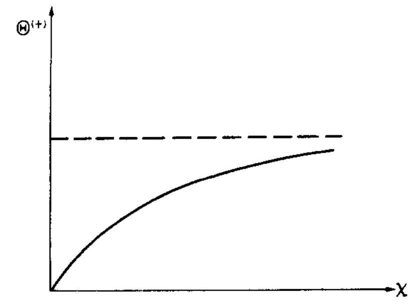

FIG. 1. Temperature distribution along the axis x for radiation beam in a co-current flow, $\Theta^{(-)}$.

$$\Theta^{(-)} = \frac{I_0\{\exp(-Pe^{(-)}\alpha x) - \exp(-\alpha x)\}}{\alpha\lambda(1 - Pe^{(-)})}, \quad (19)$$

where $\Theta^{(+)}$ and $\Theta^{(-)}$ correspond to the velocities $V^{(+)}$ and $V^{(-)}$, $Pe^{(+)} = V^{(+)}/a\alpha$, $Pe^{(-)} = V^{(-)}/a\alpha$ and $I^{(+)} = I^{(-)} = I_0\exp(-\alpha x)$.

Equation (18) has been derived with the allowance that $\Theta^{(+)}(x = 0) = 0$, $\mathrm{d}\Theta^{(+)}/\mathrm{d}x \to 0$ at $x \to \infty$. In deriving (19), it has been taken into account that $\Theta^{(-)}$, $\mathrm{d}\Theta^{(-)}/\mathrm{d}x \to 0$ at $x \to \infty$. The radiation source is located at section $x = 0$.

At $V = V^{(+)}$, the function $\Theta^{(+)}(x)$ changes monotonically, $\Theta^{(+)}$ increases due to conductive and convective heating asymptotically approaching the limit $\Theta_0^{(+)} = I_0/\lambda\alpha(1 + Pe^{(+)})$.

At $V = V^{(-)}$, the function $\Theta^{(-)}(x)$ does not change monotonically, since the heat which was absorbed is transferred by convection in the direction opposite to radiation. The temperature maximum is reached at section $x = x_m = [\alpha(Pe^{(-)} - 1)]^{-1}\ln Pe^{(-)}$ and shifts to the left with increasing $Pe^{(-)}$. Maximum temperature increases with $Pe^{(-)}$.

At $Pe^{(-)} = 1$, equation (18) acquires a singularity. On proceeding to the limit $Pe^{(-)} \to 1$, we obtain $\Theta^{(+)} = I_0 x/\lambda\exp(-\alpha x)$. The maximum in temperature distribution at $Pe^{(-)} = 1$ is attained at $x = x_m = \alpha^{-1}$.

(B) *Three-dimensional problem*

Consider a three-dimensional case for relatively short distances along the axis x, when the length of the beam path, Λ, meets the condition $\Lambda_\perp \ll \Lambda \ll \alpha^{-1}$, leaving no time for radiation to be absorbed. Two

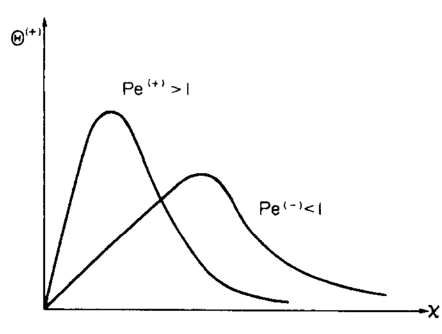

FIG. 2. Temperature distribution along the axis x for radiation beam in a counter-current flow, $\Theta^{(-)}$.

competing heat transfer processes occur: convective heat transfer along the beam and transverse conductive heating of the medium. With the latter process being predominant, the case reduces to the problem discussed earlier [5] and is not considered here. In the other limiting case, when the time of convective transfer of heat along the beam is much less than the time of conductive heating of the medium in cross-wise direction ($\Lambda/V \ll \Lambda_\perp^2/a$, which can be rewritten as $Pe = V\Lambda_\perp/a \gg \Lambda/\Lambda_\perp$), the situation reverses. The heat conduction equation (15) yields

$$V\partial_x\Theta - a\partial_{xx}^2\Theta = \frac{\alpha I}{\rho c_p}, \quad (20)$$

where I is determined by solving the system of equations (16) and (17).

Temperature distribution along the axis x will be of the same nature as in the one-dimensional case, while in cross-wise direction it will be determined, for each assigned x, by the distribution of I at this section. The nonuniformity of I (x = const, y, z) causes the nonuniformity Θ (and of ε) across the beam which results in defocusing or self-focusing of the latter. The beams the intensity (and temperature) of which diminishes away from the beam axis, e.g. the Gaussian beams (8), will defocus, while those with the intensity decreasing towards the axis, e.g. circular beams (9), will self-focus at the center and defocus at the edges. This case is formally analogous to the "banana" focusing of radiation pulses in a quiescent medium [12–14] (see also Section 4 of this paper). The difference is that now stationary beams are focused, the maximum of ε (minimum of Θ) on the beam axis is ensured by convection along the beam which prevents temperature equilibration by heat conduction in cross-wise direction, thus making for stability of this type of self-focusing in contrast to the "banana" one.

Let us consider the characteristic parameters for the focusing of circular beams and defocusing of Gaussian beams. The condition at which the divergence of radiation due to refraction caused by nonuniform heating exceeds the diffractional divergence of the beam is, by equation (16), of the form

$$\Theta \gtrsim \varepsilon_0/|\varepsilon_T|k^2\Lambda_\perp^2. \quad (21)$$

By using (18) or (20) one can obtain from (21) the characteristic intensity I_{cr} above which the diffraction can be neglected and thermal interaction makes itself felt:

$$I_{cr} = \lambda\alpha Pe^{(+)}\varepsilon_0/|\varepsilon_T|k^2\Lambda_\perp^2, \quad (22)$$

where it is taken into account that $Pe^{(+)} \gg 1$ and the path length $\Lambda \ll \alpha$. At $V = V^{(-)}$, a similar value of I_{cr} is obtained but $Pe^{(+)}$ should be substituted by $Pe^{(-)}$.

Note that the corresponding threshold power output, $N_{cr} = I_{cr}\pi\Lambda_\perp^2$, is independent of the beam width and therefore the effect of thermal defocusing (focusing) can be controlled (amplified or reduced) by varying the flow velocity at the power prescribed.

Consider focusing of circular beams as an example illustrating the concentration of output power. In the most interesting case, when $I \gg I_{cr}$, diffraction can be neglected and the angle of beam convergence can be found from equation (14). The beam focuses at the distance

$$\Lambda_{\parallel} = \Lambda_{\perp}(|\varepsilon_T|\Theta\varepsilon_0^{-1})^{-1/2} \tag{23}$$

from the entrance to the absorbing medium. Self-focusing will occur until there is a maximum on the axis in the intensity (temperature) distribution in the cross-section. Equilibration of temperature (intensity) in the central portion of the beam will take place at

$$\Lambda_{\parallel}/V \approx R^2/a, \tag{24}$$

where R is the transverse dimension of nonuniformity of I (the focal spot radius) in the cross-section $x = \Lambda_{\parallel}$, $R \ll \Lambda_{\perp}$. The value of R is determined from (23) and (24). The cross-section-average radiation intensity in the focused part of the beam is found from equation (17) (the output power balance in the beam cross-section):

$$\bar{I}(x = \Lambda_{\parallel}) = \bar{I}(x = 0)\Lambda_{\perp}^2 R^{-2}, \tag{25}$$

the factor $\exp(-\alpha\Lambda_{\parallel})$ on the RHS of (25) being disregarded since $\alpha\Lambda_{\parallel} \ll 1$. At $x > \Lambda_{\parallel}$, radiation intensity and temperature diminish towards the beam edges and the beam defocuses.

In a forced flow of weakly absorbing gases $V \gg \alpha a$ and, hence, for example, for a co-current flow, when $V = V^{(+)}$, equation (18) or (20) and (23) yield the length of self-focusing as

$$\Lambda_{\parallel} = \Lambda_{\perp}\{V + \rho c_p\varepsilon_0/|\varepsilon_T|\bar{I}(x = 0)\}^{1/2}. \tag{26}$$

Having found $\Lambda_{\parallel}$ from equation (26), one can now determine the focal spot dimension, R, from (24) and the radiation intensity at the focus from (25). The focal spot temperature is $\Theta = \bar{I}(x = \Lambda_{\parallel})/\rho c_p V$.

(C) *Discussion of results*

It has been shown earlier that stationary radiation in a quiescent medium defocuses at any intensity profile of the latter [14, 15]. The medium moving across the beam can ensure its focusing at any intensity profile [5], but the beam deflects from its line of travel and splits into several beams [9] thus reducing the efficiency of energy transfer. In the earlier discussed example it has been shown that longitudinal convection of heat can ensure stable focusing of stationary beams with circular intensity profile. The estimates of the parameters of such thermal focusing (critical power, focusing length, beam size at the focus, focal intensity and temperature) have been given. The self-focusing can be controlled by varying flow velocity, radiation intensity, etc.

Let us consider the range of applicability (the domain) of the results obtained and check them. The lower limit of velocities at which the effect is observed is determined by the rate of radiation-induced photo-absorption convection, i.e. it is required that the following inequality be satisfied [2, 6, 7]

$$V \gg V_{ph} \simeq \sqrt[3]{(\alpha I\Lambda_{\perp}^2\beta g/\rho c_p)},$$

with V being always much greater than $\alpha a (V \gg \alpha a)$.

The validity of equation (26) follows from comparison of the heat conduction equation $\rho c_p V\partial_x\Theta = \alpha I$ with equation (28) which describes thermal interaction of radiation pulses with the gas. Then, formal substitution $V \to (\alpha t)^{-1}$ transforms equation (26) into (40) which has been verified experimentally in [12].

The above effect of stationary self-focusing of circular beams is observed both in counter- and co-current gas flows provided that the moving gas path length is on the order of $\Lambda_{\parallel} \ll \alpha^{-1}$.

What has just been said is also true of quasi-stationary radiation, i.e. radiation with duration of pulses $\tau_i \gg \Lambda_{\parallel}/V$ or series of pulses with duration $\tau_i \gtrsim \Lambda_{\perp}/c_s$ (c_s is the speed of sound) and the pulse repetition rate (on-off time ratio) $\tau_r \ll \Lambda_{\parallel}/V$.

The edges of a circular beam will defocus and a bright spot of radius R surrounded by a halo will be seen at the focus, i.e. a picture similar to that observed during "banana" self-focusing [13].

Let us illustrate the above by a numerical example for a circular radiation beam of a CO_2-laser (the wavelength 10.6 μm) in the atmospheric air with $\alpha = 2.10^{-4}\, m^{-1}$. For a circular radiation beam with $\bar{I}(x = 0) = 10^6\, W/m^2$, $\Lambda_{\perp} = 10^{-1}$ m, $V = V^{(+)} = 10\,\mathrm{m\cdot s^{-1}}$, one obtains $\Lambda_{\parallel} = 10$ m, $R = 4\cdot 10^{-3}$ m, $\bar{I}(x = \Lambda_{\parallel}) = 6\cdot 10^8\,\mathrm{W/m^2}$. This estimate proves the occurrence of the effect.

4. RADIATION PULSES IN A QUIESCENT HOMOGENEOUS MEDIUM

At the initial stage of heating of a quiescent gas by a continuous radiation or by radiation pulses when the time of interaction with the medium t, $\tau_i \ll \Lambda_{\perp}^2/a$ (the corresponding Fourier number being $Fo = ta/\Lambda_{\perp}^2 \ll 1$), the heat is localized within the beam boundaries. The terms which define the heat and momentum diffusion as well as the convective terms in equations (3)–(5) are small so that the temperature and velocity of the medium satisfy the following equations [5, 7]:

$$\partial_t \mathbf{V} = \mathbf{g}\beta\Theta,\ \mathrm{div}\,\mathbf{V} = 0, \tag{27}$$

$$\rho c_p \partial_t\Theta = \alpha I, \tag{28}$$

with $\mathbf{V}(t = 0) = 0$, $\Theta(t = 0) = 0$, while the radiation intensity is determined from the solution of equations (16) and (17). The solution of (28) is

$$\Theta = \frac{\alpha}{\rho c_p}\int_0^t I(x, y, z, t)\mathrm{d}t. \tag{29}$$

The solution of the problem of thermal interaction of radiation with a gas reduces to investigation of equations (16), (17) and (29).

In equations (16) and (17), the time t is a parameter of the problem, and radiation intensity can be represented as

$$I = I_S(x, y, z)f(t), \tag{30}$$

where the dimensionless function $f(t)$ determines the radiation pulse shape in time.

By virtue of equations (29) and (30), the nonuniformity of dielectric permittivity of the medium acquires the form

$$\varepsilon_T \Theta = \frac{\varepsilon_T \alpha I_S}{\rho c_p} \int_0^t f(t)\mathrm{d}t. \tag{31}$$

Since $f(t) > 0$, the integral in (31) is always positive. A change of radiation in time characterized by the function $f(t)$ is weakly dependent on the form of the function proper and is determined by the integral of this function, i.e. the effect of thermal interaction is accumulated in time and is of integral nature. As is evident from equation (31), the choice of a spatial distribution of I_S exerts a substantial effect on the character of thermal interaction. Thus, the Gaussian beams having the profile (8) will defocus. Circular beams with the profile (9) will focus at the center where the temperature is lower and defocus at the edges.

This is the so-called "banana" self-focusing phenomenon [5, 12–14].

Let us consider the results that follow from the analysis of a corresponding linearized problem.

In the one-dimensional case of an infinitely wide radiation beam, when all of the variables depend on x alone, the problem admits an exact solution. Investigation of the stability of the solution with respect to small spatial disturbances of I, S and Θ allows one to obtain critical (theshold) intensities when heating of the medium controls propagation of radiation. Reduction in radiation intensity of defocusing beams is the result of the effect of two factors: divergence of the beam due to thermal defocusing produced by heating and absorption of radiation, $I \propto \exp(-\alpha x)$. The linearized theory makes it possible to obtain the characteristic parameters of the problem. Thermal interaction effects are especially pronounced when

$$I \gtrsim I_{cr_2} = \frac{\alpha \Lambda_\perp^2 \rho c_p \varepsilon_0}{2|\varepsilon_T| \int_0^t f(t)\mathrm{d}t} \tag{32}$$

and become weaker over the distances

$$x > x_2 = \alpha^{-1} \ln\{2|\varepsilon_T| I_0 \int_0^t f(t)\mathrm{d}t/\alpha\Lambda_\perp \rho c_p \varepsilon_0\}. \tag{33}$$

The angle of beam divergence due to heating of the medium is determined from equations (14) and (31)

$$\varphi = (\varepsilon_0 \rho c_p/|\varepsilon_T| \alpha I_0 \int_0^t f(t)\mathrm{d}t)^{-1/2}. \tag{34}$$

(A) "*Banana*" *self-focusing*

Let us consider an interesting and practically useful case of "banana" self-focusing of radiation pulses having a circular intensity profile (9). We shall investigate the process of thermal interaction of radiation pulses and the initial stage of propagation of stationary radiation beams when

$$f(t) = \begin{cases} 1, & 0 < t < \tau_i \\ 0, & \tau_i > t,\ t < 0 \end{cases} \quad \text{or}\ f(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}. \tag{35}$$

For mean values of velocity and temperature with respect to the beam cross-section (we shall limit ourselves to horizontal orientation of radiation only), we obtain from (27), (28) and (35)

$$\bar{\Theta}(x, t) = \alpha \bar{I}_S(x)t/\rho c_p,$$
$$\bar{V}(x, t) = g\beta\alpha \bar{I}_S(x)t^2/\rho c_p. \tag{37}$$

The cross-section-mean intensities in the central focusing part of the beam in the initial cross-section and at the focal spot are similarly found from (25)

$$\bar{I}_S(x = \Lambda_\parallel) = \bar{I}_S(x = 0)\Lambda_\perp^2 R^{-2}. \tag{38}$$

Here we assume that the intensities $\bar{I}(x = 0)$ are so large that $\Lambda_\parallel \ll x_2$, the quantity R characteristizes the beam radius at the focus.

Focusing terminates and at $x > \Lambda_\parallel$ gives place to defocusing when the transverse dimension of the beam, R, becomes so small that heat diffusion (conduction) or convection succeed in equilibrating the temperature in the beam cross-section, i.e.

$$\min\left\{\frac{R^2}{a}, \frac{R}{\bar{V}}\right\} \approx t. \tag{39}$$

The condition (39) can be rewritten as $\max(Fo, Ho) = 1$, with the Fourier number being $Fo = ta/R^2$ and the homochromaticity number, $Ho = t\bar{V}/R$.

Equations (36)–(39) form a closed system for investigation of the "banana" self-focusing of the central portion of a circular beam, the focusing length $\Lambda_\parallel$ being determined, by virtue of (34), from

$$\Lambda_\parallel = \Lambda_\perp \left(\varepsilon_0 \rho c_p/|\varepsilon_T| \alpha \bar{I}_S(x = 0) \int_0^t f(t)\mathrm{d}t\right)^{1/2}. \tag{40}$$

The photoabsorption convection velocity $\bar{V}$ depends on radiation intensity and time ($\bar{V} \propto \bar{I}_S t^2$). Therefore, at large enough radiation intensities and/or times t, $Ho > Fo$ and the constraint on focusing (dimensions and, hence, intensity) will be determined by convection, while at small ones, by heat conduction. By (39), the focus dimensions increase with time, $R = R(t, \bar{I}_S)$, and the focus moves towards radiation [by (40), the value of $\Lambda_\parallel$ decreases] at the speed $V = \mathrm{d}\Lambda_\parallel/\mathrm{d}t \propto t^{-1/2}$. When $R = \Lambda_\perp$, focusing terminates and in subsequent instants of time defocusing is observed.

At the initial instant, the radiation-induced flow velocity is small, $R \approx \sqrt{at}$, and the intensity at the focus is

$$\bar{I}_S(x = \Lambda_\parallel) = \Lambda_\perp^2 \bar{I}_S(x = 0)(at)^{-1}. \tag{41}$$

In the case of large intensities or times of radiation interaction with the gas, when the velocity of convection increases and becomes equal to the rate of heat diffusion across the focus ($\bar{V} = a/R = \sqrt{a/t}$) the intensity at the focus is determined, by virtue of (37) and (39), as

$$\bar{I}_S^3(x = \Lambda_\parallel) = \Lambda_\perp^2 I(x = 0)\left(\frac{\rho c_p}{g\beta\alpha t^3}\right)^2 \quad (42)$$

and $R \propto t$.

Thus, the focus, while expanding, moves towards the beam. The "banana" self-focusing terminates and gives place to defocusing at $R \geqslant \Lambda_\perp$, i.e. at $t \geqslant \Lambda_\perp/\bar{V}(x = \Lambda_\parallel)$. The temperature at the beam focus is determined from equation (36).

(B) *Discussion of results*

One of the first studies of thermal interaction of radiation pulses with substance appeared in 1966 [18]. Since then, a vast number of theoretical and experimental investigations have been carried out [2, 4, 5, 12–14], but the problem of such interaction has not been elucidated completely. Not all of the features and parameters of the process have been determined. Usually, a single factor, namely attenuation of radiation due to thermal defocusing, is taken into account, while it follows from (32) that in the case of rather short radiation pulses and broad beams, both thermal defocusing and reduction in the intensity due to radiation absorption should be allowed for. Defocusing can be disregarded at $x > x_2$.

The threshold intensity of the radiation beam, $N_{cr} = \pi \cdot \Lambda_\perp^2 \cdot I_{cr}$ where

$$I_{cr} = \varepsilon_0\rho c_p/2\alpha|\varepsilon_T| \int_0^t f(t)\mathrm{d}t k^2\Lambda_\perp^2$$

is the intensity above which diffractive divergence of the beam can be neglected compared with the heating-induced refraction, is independent of the beam width as is also the divergence (convergence) angle of the beam, φ [see equation (34)]. For all beams of the given intensity, $N > N_{cr}$, defocusing is identical. In the atmospheric air, for radiation with the wavelength $\lambda_0 \simeq 5 \cdot 10^{-5}$ m and $\alpha = 10^{-4}\,\mathrm{m}^{-1}$, $N_{cr} = 1 \cdot t^{-1}$(W).

Quenching of thermal defocusing of radiation pulses by selection of the beam width is impossible. Nevertheless, there is a possibility to compensate for thermal blooming of pulses both by means of pre-focusing of the beam (which is discussed in detail in [5, 11]) and by choosing a circular profile of the beam intensity.

A circular radiation beam has the temperature minimum in its central portion which ensures its thermal self-focusing. The "banana" self-focusing effect was first predicted and detected experimentally by the authors of [12, 13] when a luminescent central portion of the beam was registered against the background of an obscured halo produced by defocusing of the beam edges. The focus, while moving, left a luminescent trace. In [5, 12–14], the focus distances for "banana" self-focusing and the products of the dynamics of focusing relaxation due to heat conduction have been determined.

In the present paper, radiation intensity at the focus and the size of the latter are shown to be limited at the initial instant of time at small intensities by transverse heat conduction. With the intensities in the beam being large, the focus dimensions and intensity are limited by radiation-induced convection. In the latter case the lifetime of the "banana" self-vocusing is shorter. Radiation intensities at the focus can increase by several orders of magnitude as compared with the incident pulse intensity.

The "banana" self-focusing is highly unstable. The mechanisms limiting the constraint on the intensity at the focus are characterized by very small velocities, on the order of a/R. That is why a medium moving across the beam at a very high velocity rapidly equilibrates the temperature distribution and the focusing effect disappears. This conclusion agrees with the experimental results [13].

5. RADIATION PULSES IN NONUNIFORMLY HEATED GASES

Let us consider the effect of heat transfer on propagation of high-power radiation pulses in moving gases. The presence of radiation contributes additionally to nonuniformity of the flow temperature distribution. The temperature nonuniformity is determined by the sum $\vartheta + \Theta$, it develops due to the temperature nonuniformity resulting from heat transfer in the absence of radiation and that associated with absorption of radiation. Solution of this problem is of interest for studying high-power radiation propagation in gas lenses. In gas lenses, nonuniformity of the temperature field of a heated gas develops which imparts focusing (lens-like) properties to the gas [19]. The presence of high-power radiation changes these focusing properties.

When the velocity of the forced external gas flow $|\mathbf{V}| \gg |\mathbf{V}_{ph}|$, where $V_{ph} \simeq g\beta\alpha I t^2/\rho c_p$ is the radiation-induced convection velocity, the effect of radiation-induced flow on the gas stream can be neglected and the Navier–Stokes equations (3) and (4) will acquire the form

$$\partial_t\mathbf{V} + (\mathbf{V}\nabla)\mathbf{V} = -\frac{\nabla P}{\rho} + \nu\nabla^2\mathbf{V},\ \mathrm{div}\,\mathbf{V} = 0. \quad (43)$$

When the time of gas interaction with radiation $t \ll \Lambda_\perp/V$, $\Lambda_\perp^2/a_T$ (the corresponding Fourier and homochromaticity numbers Fo, $Ho \ll 1$), the heat-conduction equation (5) yields two equations for the temperature ϑ and Θ:

$$\rho c_p\partial_t\vartheta + \mathbf{V}\nabla\vartheta = \lambda\nabla^2\vartheta, \quad (44)$$

$$\rho c_p\partial_t\Theta = \alpha I, \quad (45)$$

in which the radiation intensity is found from (11)–(13). The solution of the problem of (43) and (44) with the corresponding initial and boundary conditions is assumed to be known.

Now determine the conditions required in order that the highpower pulses with the circular and

Gaussian intensity profiles should focus. Consider a two-dimensional symmetric radiation beam (when all of the variables depend on x and y only, with nothing changing in the direction z) and a corresponding symmetric focusing temperature distribution ϑ. Equations (11) and (12) in the approximation of geometrical optics will take on the form

$$2\partial_x S + (\partial_y S)^2 = \frac{\varepsilon_T(\vartheta + \Theta)}{\varepsilon_0}, \quad (46)$$

$$\partial_x\psi^2 + \partial_y\psi^2\partial_y S + \psi^2\partial_{yy}S + \alpha\varphi^2 = 0. \quad (47)$$

Now the function ϑ should be determined from the condition of radiation focusing and Θ, from equation (45).

On integrating (45) and introducing the notation

$$\varepsilon_2 = \frac{\varepsilon_T c\sqrt{\varepsilon_0}\alpha}{8\pi\rho c_p}\int_0^t f(t)\mathrm{d}t < 0, \quad (48)$$

rewrite equation (46) in the form

$$2\partial_x S + (\partial_y S)^2 = \frac{\varepsilon_2\psi^2}{\varepsilon_0} + \frac{\varepsilon_T\vartheta}{\varepsilon_0}. \quad (49)$$

Thus, the problem of radiation behaviour in a nonuniformly heated medium has reduced to integration of (47) and (49). This system of equations reduces to the system (49) and to

$$\frac{\mathrm{d}\ln\psi^2}{\mathrm{d}x} = -\frac{\varepsilon_2}{2\varepsilon_0}\partial_y\int\partial_y\psi^2\,\mathrm{d}x - \frac{\varepsilon_T}{2\varepsilon_0}\partial_y\vartheta\,\mathrm{d}x - \alpha \quad (50)$$

In the general case, construction of solution for (49) and (50) is impossible. However, it is already evident from (50) in which way propagation of beams is affected by heating of the medium due to radiation absorption [first term on the RHS of (50)], by the temperature field of the medium [second term on the RHS of (50)] and by reduction of radiation due to absorption (third term). Contribution of these factors to diminishing of the amplitude logarithm (the so-called "level") turns out to be an additive one, hence it is easy to determine from (50) the gradient $\partial_y\vartheta$ which ensures radiation focusing and thus compensates for the intensity losses due to thermal interaction of radiation with the medium.

When one and the same gas is used as a working medium of a gas lens and a surrounding medium, then compensation for defocusing and absorption of the Gaussian beams on the path of the length L_1 requires a focusing element (gas lens) of length $L_2 \leqslant L_1$ with the temperature gradient satisfying the condition

$$\partial_y\phi \gtrsim \left|\left(\frac{\varepsilon_2\psi_0^2}{\varepsilon_T\Lambda_\perp} + \frac{2\Lambda_\perp\alpha\varepsilon_0}{L_1\varepsilon_T}\right)\frac{L_1}{L_2}\right|. \quad (51)$$

For circular beams with the same parameters the condition imposed on $\partial_y\vartheta$ is less rigid

$$\partial_y\vartheta \gtrsim \left|\left(\frac{\varepsilon_2\psi_0^2}{\varepsilon_T\Lambda_\perp} - \frac{\Lambda_\perp\alpha\varepsilon_0}{L_1\varepsilon_T}\right)\frac{L_1}{L_2}\right|, \quad \partial_y\vartheta > 0. \quad (52)$$

Corresponding equations for different media in and around a gas lens are obtained in a similar way. If the conditions (51) and (52) are not met, the use of gas lenses to focus radiation is of little value.

We shall clarify our preceding remarks using the example of operation of a gas lens the walls of which are heated by a constant heat flux [20]. Temperature distribution and a corresponding distribution of the refractive index can be presented in the form

$$n(y) = \text{const} - \beta y^2,$$

where

$$\beta = \frac{2q_w y_0 - 3\bar{q}_v y_0^2}{\lambda T_0},$$

and q_w, $\bar{q}_v$ stand for the heat flux supplied to the wall and heat release due to radiation absorption, respectively, y_0 is the gas lens radius. The focal length of the gas lens with the above refractive index distribution can be approximated as $f \approx 1/\beta L_2$.

Analysis of the above expressions shows that at

$$\bar{q}_v = \frac{2}{3}\frac{q_w}{y_0},$$

the focal length turns out to be infinite. In other words, when the light source power attains the above magnitude, thermal defocusing occurs and the efficiency of the use of the gas lens is reduced.

(A) *Discussion of results*

The criteria to be satisfied by the gas lenses in order to focus high-power radiation with Gaussian and circular intensity profiles have been determined. Related criteria follow directly from comparison of distribution of temperatures ϑ and Θ within the boundaries of the beam of the width $\Lambda_\perp$. However, in (51) and (52) intensity attenuation due to radiation absorption has also been allowed for. The criteria obtained take into account attenuation of intensity due to radiation absorption and thermal defocusing. In the case of a beam with cylindrical symmetry, an additional term, $(-\int y^{-1}d_y S\mathrm{d}x)$, appears on the RHS of equation (50).

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INTERACTION THERMIQUE DU RAYONNEMENT D'UN LASER A HAUTE PUISSANCE AVEC DES GAZ

Résumé—On étudie l'intéraction thermique du rayonnement avec un milieu immobile et avec un milieu en écoulement à co-ou-contre courant sur le parcours du rayonnement. On considère le cas d'une lentille de gaz convergente et d'une source de chaleur induite par le rayonnement qui contribue à la non-uniformité de la température. On étudie les conditions d'auto-convergence de rayons stationnaires gaussiens et circulaires dont l'intensité décroit à partir de l'axe. On considère la dynamique des mécanismes thermiques et optiques dans le cas d'une auto-convergence en forme de banane.

THERMISCHE WECHSELWIRKUNG VON HOCHLEISTUNGS-LASER-STRAHLUNG MIT GASEN

Zusammenfassung—Diese Arbeit befaßt sich mit der Untersuchung der thermischen Wechselwirkung von Strahlung mit einem ruhenden Medium und einem in Gleich- und Gegenrichtung strömenden Medium. Es wird der Fall betrachtet, daß durch den Betrieb eines fokussierenden Gas-Linsen-Lasers die Erwärmung durch Strahlung die Ungleichmässigkeit des Temperaturfeldes zusätzlich erhöht. Es werden die Bedingungen untersucht, die zur Selbstfokussierung stationärer Gauss'scher und zirkularer Strahlen nötig sind; die Intensität dieser Strahlen nimmt gegen den Strahlmittelpunkt ab. Die Dynamik der thermischen und optischen Prozesse im Fall der "Bananen"—Selbstfokussierung ist berücksichtigt.

ТЕПЛОВОЕ ВЗАИМОДЕЙСТВИЕ ЛАЗЕРНОГО ИЗЛУЧЕНИЯ СО СРЕДОЙ

Аннотация — Исследуется тепловое взаимодействие излучения с покоящейся средой, а также при спутном и встречном вынужденном движении среды на трассе распространения излучения. Рассмотрена работа газовой линзы при фокусировке мощного лазерного излучения в случае, когда тепловыделение за счет поглощения излучения вносит дополнительный вклад в неоднородность распределения температуры. Исследованы условия самофокусировки для стационарных гауссовых и кольцевых пучков излучения, интенсивность которых уменьшается к центру пучка. Рассматривается динамика тепловых и оптических процессов при «банановой» самофокусировке.